AP Calculus Summer Homework

Name ___________________________________

Dear Future Calculus Student,

I hope you are excited for your upcoming year of AP Calculus! This branch of mathematics is extremely exciting and is unlike any other branch of mathematics you've studied thus far. In a nutshell, calculus is described as 'the mathematics of change' – how fast things change, how to predict change, and how to use information about change to interpret the world around us. As is true with each new branch of mathematics, calculus takes what you already know a step further.

Going into AP Calculus, there are certain algebraic skills that have been taught to you over the previous years, which we must assume you have. If you do not have these skills, you will find that you will consistently get problems incorrect next year, even though you understand the calculus concepts. As you can imagine, it is extremely frustrating for students to be tripped up by the algebra and not the calculus. This summer homework is intended to help you review and/or get reacquainted with the algebra concepts needed to be successful in calculus.

Please submit this packet during your **second** calculus class period this fall. **It will be graded.** Work needs to be shown in a neat and organized manner, and it is perfectly acceptable to complete the packet on separate sheets of paper. Just be sure to staple any extra papers to the packet. Also, **do not** rely on a calculator. Half of your AP exam next year will be taken without a calculator; paper and **pencil** techniques only.

Need help with your Summer math packet???

Assistance will be available Monday August 24th through Thursday August 27th: Mrs. Driscoll will be holding office hours from 12:00 p.m. to 2 p.m. daily via zoom. (You will be invited to be part of the Google Classroom for AP Calculus over the summer where more information on how to join zoom office hours will be posted.) Additionally, you may email your questions to Mrs. Driscoll at ndriscoll@wareham.k12.ma.us or to Mrs. Medina at mmedina@wareham.k12.ma.us. To ensure the fastest response, please include your name, summer assignment name, and (if possible) a picture of the problem and your accompanying work.

Directions:

- Before answering any questions, read through the given notes and examples for each topic.
- This packet is to be submitted during your **second** calculus class period.
- All work must be shown in the packet or on a separate sheet of paper stapled to the packet.
- **To avoid a penalty on your grade, final answers MUST BE BOXED or CIRCLED.**
- We will have a ***quiz on the unit circle*** during the second week of school. I recommend that you study the unit circle over the summer.

Part 1 - Functions:

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$

1. $f(2) =$	2. $g(-3) =$
3. $f(t + 1) =$	4. $f[g(-2)] =$
5. $g[f(m + 2)] =$	6. $[f(x)]^2 - 2g(x) =$

Let f(x) = sin(2x). Find each exactly.

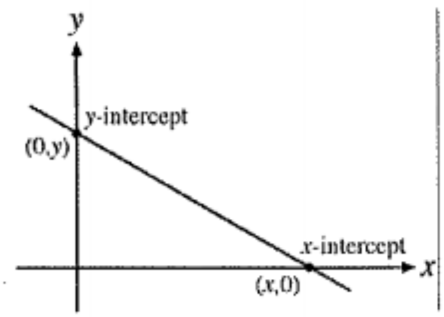
7. $f(\frac{\pi}{4})$ =	8. $f(\frac{2\pi}{3})$ =

Let $f(x) = x^2$, $g(x) = 2x + 5$, $h(x) = x^2 - 1$.

9. h[f(-2)] =	10. f[g(x – 1)] =
11. g[h(x^3)] =	

Part 2 - Intercepts of a Graph

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

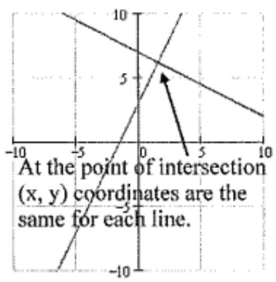
$\underline{x-\text{int. } (Let\ y=0)}$	$\underline{y-\text{int. } (Let\ x=0)}$
$0 = x^2 - 2x - 3$	$y = 0^2 - 2(0) - 3$
$0 = (x-3)(x+1)$	$y = -3$
$x = -1\ or\ x = 3$	$y-$intercept $(0,-3)$
$x-$intercepts $(-1,0)$ *and* $(3,0)$	

Find the x- and y-intercepts for each of the following.

12. $y = 2x - 5$	13. $y = x^2 + x - 2$
14. $y = x\sqrt{16 - x^2}$	15. $y^2 = x^3 - 4x$

Part 3 - Points of Intersection

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP
Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2^{nd} Trace) and hit INTERSECT.

Example: Find all points of intersection of $\begin{array}{l} x^2 - y = 3 \\ x - y = 1 \end{array}$

ELIMINATION METHOD

Subtract to eliminate y

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

Plug in $x = 2$ and $x = -1$ to find y

Points of Intersection: $(2,1)$ and $(-1,-2)$

SUBSTITUTION METHOD

Solve one equation for one variable.

$y = x^2 - 3$

$y = x - 1$

Therefore by substitution $x^2 - 3 = x - 1$

$x^2 - x - 2 = 0$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $\begin{cases} x + y = 8 \\ 4x - y = 7 \end{cases}$

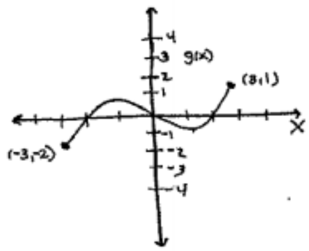
17. $\begin{cases} x^2 + y = 6 \\ x + y = 4 \end{cases}$

18. $\begin{cases} x = 3 - y^2 \\ y = x - 1 \end{cases}$

Part 4 - Domain and Range

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).

The domain is the set of inputs of the function. Input values run along the horizontal axis. The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is $[-3,3]$, that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is $[-2,1]$, all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN

For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

RANGE

The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

Find the domain and range of each function. Write your answer in interval notation.

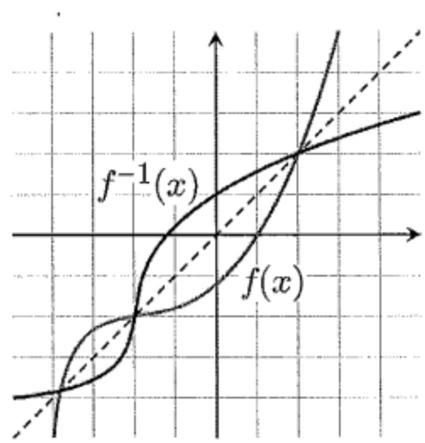
19. $f(x) = x^2 - 5$	20. $f(x) = -\sqrt{x+3}$
21. $f(x) = 3\sin x$	22. $f(x) = \frac{2}{x-1}$

Part 5 - Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse for each function.

23. $f(x) = 2x + 1$	24. $f(x) = \frac{x^2}{3}$
25. $g(x) = \frac{5}{x-2}$	26. $y = \sqrt{4-x} + 1$

27. If the graph of $f(x)$ has the point (2, 7), then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

Part 6 - Equation of a Line

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of $\frac{2}{3}$.

32. Use point-slope form to find the equation of the line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

33. Use point-slope form to find a line perpendicular to y = -2x + 9 passing through the point (4, 7).

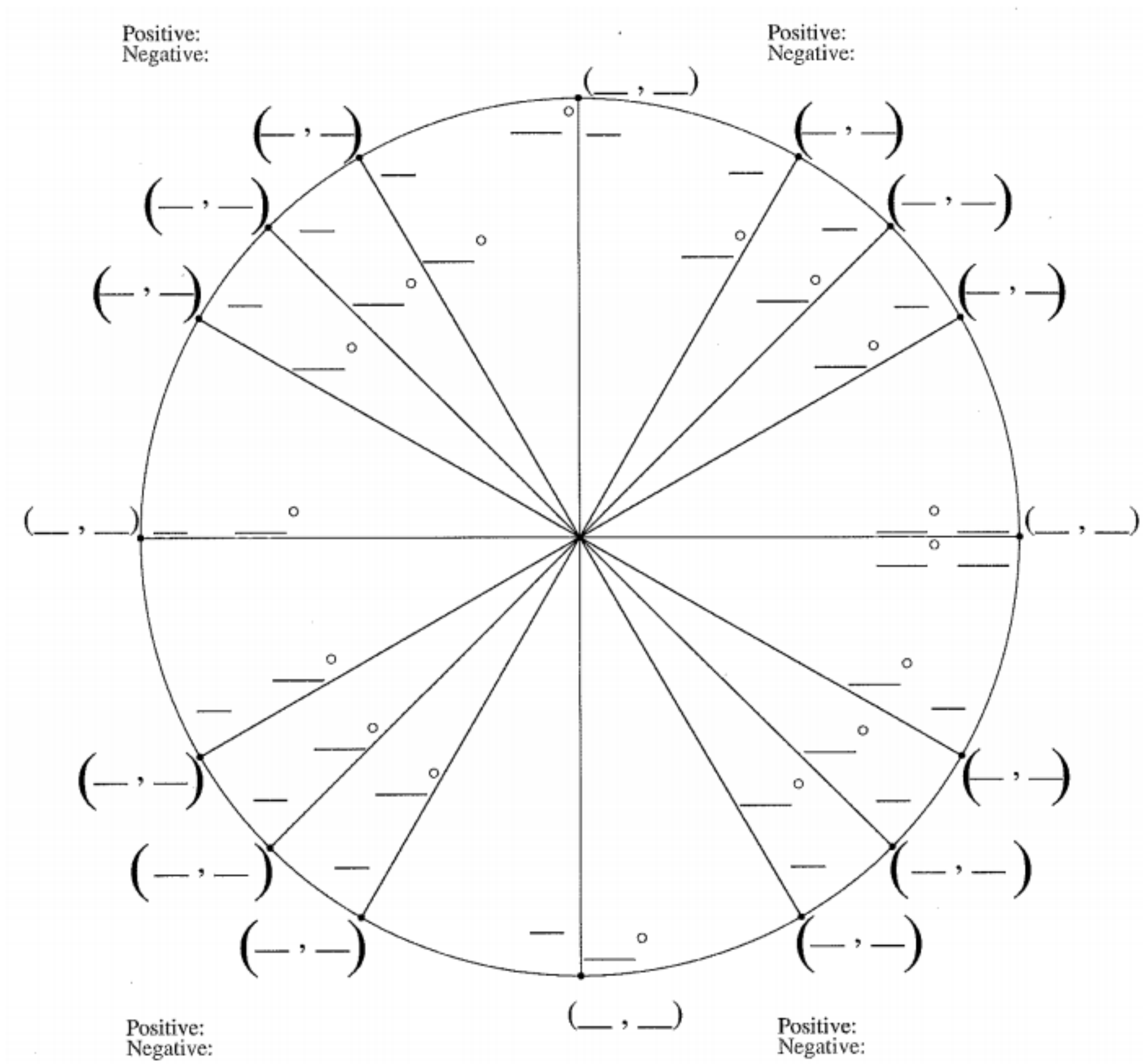
34. Find the equation of the line passing through the points (-3, 6) and (1, 2).

35. Find the equation of the line with an x-intercept (2, 0) and a y-intercept (0, 3).

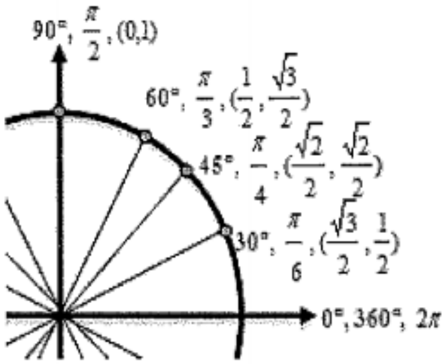
Part 7 – Unit Circle

Fill in the unit circle below with the appropriate exact values (degrees and radians).

The Unit Circle



Part 8 - Unit Circle



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2} = 1$ $\quad\cos\frac{\pi}{2} = 0$ $\quad\tan\frac{\pi}{2} = und$

36. You must have these memorized or know how to calculate their values without the use of a calculator.

a. $\sin\pi =$	b. $\cos\frac{3\pi}{2} =$	c. $\sin\left(-\frac{\pi}{2}\right) =$	d. $\sin(\frac{5\pi}{4}) =$
e. $\cos\frac{\pi}{4} =$	f. $\cos(-\pi) =$	g. $\cos\frac{\pi}{3} =$	h. $\sin(\frac{5\pi}{6}) =$
i. $\cos\frac{2\pi}{3} =$	j. $\tan\frac{\pi}{4} =$	k. $\tan\pi =$	l. $\tan\frac{\pi}{3} =$
m. $\cos\frac{4\pi}{3} =$	n. $\sin(\frac{11\pi}{6}) =$	o. $\tan\frac{7\pi}{4} =$	p. $\sin\left(-\frac{\pi}{6}\right) =$

Part 9 - Trigonometric Equations

Solve each of the equations for $0 \leq x < 2\pi$.

37. $\sin x = -\frac{1}{2}$	38. $2\cos x = \sqrt{3}$
39. $4sin^2(x) = 3$ *Recall $sin^2(x) = (sinx)^2$ *Recall if $x^2 = 25$, then $x = \pm 5$.	40. $2cos^2(x) - 1 - cosx = 0$

Part 10 - Transformation of Functions

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How does the graph of g(x) differ from the graph of f(x)?

42. Write an equation for the function that has the shape of $f(x) = x^3$, but moved 6 units to the left and reflected over the x-axis.

43. If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

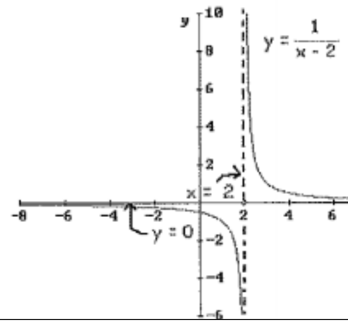
a. $f(x) - 3$	b. $f(x - 3)$	c. $2f(x)$	d. $f(x - 2) + 1$	e. $-f(x)$

Part 11 – Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$

Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



Find the vertical asymptote for each of the following problems:

44. $f(x) = \frac{1}{x^2}$	45. $f(x) = \frac{x^2}{x^2-4}$	46. $f(x) = \frac{2+x}{x^2(1-x)}$
47. $f(x) = \frac{4-x}{x^2-16}$	48. $f(x) = \frac{x-1}{x^2+x-2}$	49. $f(x) = \frac{5x+20}{x^2-16}$

Part 12 – Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \frac{2x^2+x-1}{3x^2+4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \frac{2x^2+x-1}{3x-3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all horizontal asymptotes in the following problems:

50. $f(x) = \frac{x^2-2x+1}{x^3+x-7}$

51. $f(x) = \frac{5x^3-2x^2+8}{4x-3x^3+5}$

52. $f(x) = \frac{4x^2}{3x^2-7}$

53. $f(x) = \frac{(2x-5)^2}{x^2-x}$

54. $f(x) = \frac{-3x+1}{\sqrt{x^2+x}}$ *Remember $\sqrt{x^2} = \pm x$

This page is VERY important for our first unit!

Part 13 – Exponential Functions

Solve for x:

Example: Solve for x

$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$	Get a common base
$2^{2x+2} = 2^{-3x+2}$	Simplify
$2x+2 = -3x+2$	Set exponents equal
$x = 0$	Solve for x

55. $3^{3x+5} = 9^{2x+1}$

56. $(\frac{1}{9})^x = 27^{2x+4}$

57. $(\frac{1}{6})^x = 216$

Part 14 – Logarithms

Evaluate the following logarithms:

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the SAME thing. **Remember: A logarithm is just an exponent!** Recall $\ln x = \log_e x$. The value of e is 2.718281828… or $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$.

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^? = 8$

Therefore $? = 3$

Thus $\log_2 8 = 3$

58. $\log_7 7 =$

59. $\log_3 27 =$

60. $\log_2 \frac{1}{32} =$

61. $\log_{25} 5 =$

62. $\log_9 1 =$

63. $\log_4 8 =$

64. $\ln\sqrt{e} =$

65. $\ln\frac{1}{e} =$

Part 15 – Properties of Logarithms

$\log_b xy = \log_b x + \log_b y$ $\quad$ $\log_b \frac{x}{y} = \log_b x - \log_b y$ $\quad$ $\log_b x^y = y \log_b x$ $\quad$ $b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use properties of logarithms to evaluate the following:

66. $\log_2 2^5$	67. $\ln e^3$	68. $\log_2 8^3$	69. $\log_3 \sqrt[5]{9}$
70. $2^{\log_2 10}$	71. $e^{\ln 8}$	72. $9 \ln e^2$	73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$

75. $\log_2 40 - \log_2 5$

76. $\log_2(\sqrt{2})^5$