IB Mathematics SL: Analysis and Approaches Summer Homework

Name ____

Dear Future IB Mathematics SL Student,

I hope you are excited for your upcoming year in IB Math SL! The purpose behind this summer homework packet is to reacquaint you with the necessary skills to be successful in this year's math course.

At first glance this packet may seem overwhelming. However, there are approximately 9 weeks of summer. Pace yourself. There are 13 Parts of this packet – complete two parts each week and you will easily be able to complete the assignment before your return to school in the fall. Please be prepared to submit this assignment during your **second IB Math SL class.** It will be graded for accuracy as well as completion. Work needs to be shown in a neat and organized manner, and it is perfectly acceptable to complete the packet on separate sheets of paper. Just be sure to staple any extra papers to the packet. Also, do not rely on a calculator!

Show ALL work for each problem and take your time. Remember, this will be your first impression to your new math teacher, and you want to make sure that it is a positive one! See below for directions and helpful websites. We hope you have a wonderful summer!

Best,

Wareham High School Math Department

Need help with your Summer math packet???

Feel free to email Mrs. Medina at mmedina@wareham.k12.ma.us with any questions you might have. To ensure the fastest response, please include your name, summer assignment name, and (if possible) a picture of the problem and your accompanying work.

Directions:

- Before answering any questions, read through the given notes and examples for each topic.
- This packet is to be submitted during your **second IB Math SL class** period.
- All work must be shown in the packet or on a separate sheet of paper stapled to the packet.
- **To avoid a penalty on your grade, final answers MUST BE BOXED or CIRCLED.**

Part 1 – Functions

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

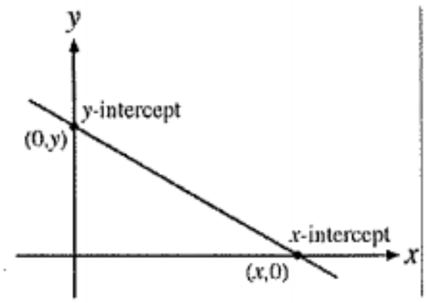
$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = x^2,\ g(x) = 2x + 5,\ h(x) = x^2 - 1$, find:

1) $h(-2)$	2) $f(g(-4))$
3) $h(x + 2)$	4) $(g(x))^2 - 2f(x)$

Part 2 – Intercepts of a Graph

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

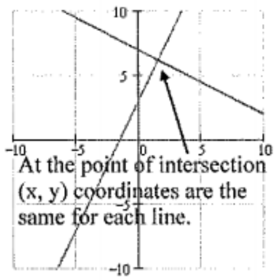
$x-$int. ($Let\ y = 0$)	$y-$int. ($Let\ x = 0$)
$0 = x^2 - 2x - 3$	$y = 0^2 - 2(0) - 3$
$0 = (x-3)(x+1)$	$y = -3$
$x = -1\ or\ x = 3$	$y-$intercept $(0,-3)$
$x-$intercepts $(-1,0)$ *and* $(3,0)$	

Find the $x -$ and $y -$intercepts for each function below.

5) $y = 2x - 8$	6) $y = x^2 + 5x + 6$
7) $y = \sqrt{x+4}$	8) $y = 3^x - 9$

Part 3 – Points of Intersection

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP
Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2nd Trace) and hit INTERSECT.

Example: Find all points of intersection of $\begin{matrix} x^2 - y = 3 \\ x - y = 1 \end{matrix}$

ELIMINATION METHOD
Subtract to eliminate y
$x^2 - x = 2$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2$ or $x = -1$
Plug in $x = 2$ and $x = -1$ to find y
Points of Intersection: $(2,1)$ and $(-1,-2)$

SUBSTITUTION METHOD
Solve one equation for one variable.
$y = x^2 - 3$
$y = x - 1$
Therefore by substitution $x^2 - 3 = x - 1$
$x^2 - x - 2 = 0$
From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations. Write each solution as an ordered pair.

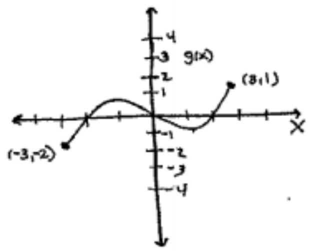
9) $\begin{cases} 4x - y = 10 \\ 2x + y = 8 \end{cases}$

10) $\begin{cases} y = 3x - 4 \\ x^2 + y = 6 \end{cases}$

Part 4 – Domain and Range

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).

The domain is the set of inputs of the function. Input values run along the horizontal axis. The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3. So Domain is [-3,3], that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is [-2,1], all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN
For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

RANGE
The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

Find the domain and range of each function. Write your answer using interval notation.

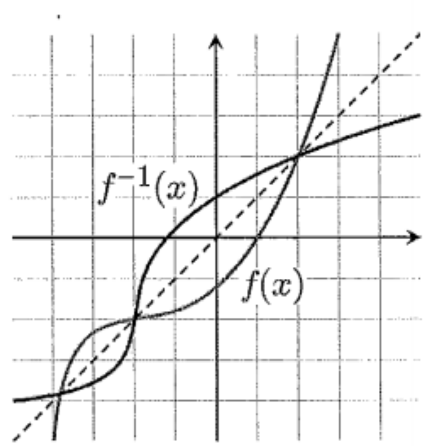
11) $f(x)=x^2+5$	12) $g(x)=\sqrt{x-3}$
13) $h(x)=\dfrac{2}{x-1}$	14) $f(x)=4^{x-2}+5$

Part 5 – Inverses

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse of each function below.

15)
$$f(x) = 2x + 1$$

16) (Domain restriction: $x \geq 0$)
$$g(x) = x^2 - 7$$

17)
$$h(x) = \frac{5}{x-2}$$

18)
$$f(x) = \sqrt{x-4} + 3$$

19) If the graph of $f(x)$ has the point $(2,\ 7)$, then what is one point that will be on the graph of $f^{-1}(x)$?

20) Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

Part 6 – Transformation of Functions

$h(x)=f(x)+c$	Vertical shift c units up	$h(x)=f(x-c)$	Horizontal shift c units right
$h(x)=f(x)-c$	Vertical shift c units down	$h(x)=f(x+c)$	Horizontal shift c units left
$h(x)=-f(x)$	Reflection over the x-axis		

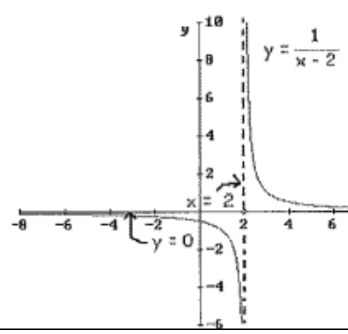
21) How is the graph of $g(x) = (x-3)^2 + 1$ related to the parent function $f(x) = x^2$? Describe the transformations that occurred from the parent function to the given function.

22) Write an equation for a function, $g(x)$, that has the shape of $f(x) = x^3$, but is translated 6 units to the left and is reflected over the x −axis.

Part 7 – Vertical Asymptotes

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \frac{1}{x-2}$
Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



Find the vertical asymptote(s) of each rational function below.

23) $f(x) = \frac{5}{x+2}$	24) $g(x) = \frac{x+3}{x^2-6x+8}$	25) $h(x) = \frac{x-7}{x^2-49}$

Part 8 – Horizontal Asymptotes

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y=\frac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y=0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y=\frac{2x^2+x-1}{3x^2+4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y=\frac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y=\frac{2x^2+x-1}{3x-3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Find the horizontal asymptote of each rational function below.

26) $f(x)=\frac{4x+10}{2x-3}$	27) $g(x)=\frac{3}{x+9}$	28) $h(x)=\frac{x^2-1}{x+2}$
29) $f(x)=\frac{x^2-2x+1}{x^3+x-7}$	30) $g(x)=\frac{-5x^3-2x^2+8}{3x^3+4x-5}$	31) $h(x)=\frac{10x^2}{2x^2-1}$

Part 9 – Exponential Equations

Solve $5^{3x-2} = 125^{2x}$

$$5^{3x-2} = 125^{2x}$$
$$5^{3x-2} = (5^3)^{2x}$$
$$5^{3x-2} = 5^{6x}$$
$$3x-2 = 6x$$
$$-2 = 3x$$
$$x = -\frac{2}{3}$$

Example: Solve for x

$$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$	Get a common base
$2^{2x+2} = 2^{-3x+2}$	Simplify
$2x+2 = -3x+2$	Set exponents equal
$x = 0$	Solve for x

Solve each exponential equation below.

32) $5^{7x-6} = 5^{-3x+24}$

33) $3^{6x-4} = 9^{2x+1}$

34) $\left(\frac{1}{6}\right)^{x} = 216$

35) $\left(\frac{1}{4}\right)^{3x} = 16^{-2x+4}$

Part 10 – Logarithms

The logarithmic equation $\log_b y = x$ can be written as an exponential equation $b^x = y$. They mean the SAME thing. You can use this fact to evaluate logarithms.

Recall the natural logarithm $\ln x = \log_e x$. The value of $e \approx 2.718281828$ ... or $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$.

Evaluate $\log_4 8$

Write an equation in logarithmic form

$\log_4 8 = x$

Convert the equation in exponential form

$4^x = 8$

Write each side with a base of 2

$(2^2)^x = 2^3$

$(2)^{2x} = 2^3$

$2x = 3$

$x = \frac{3}{2}$

So $\log_4 8 = \frac{3}{2}$

Example : Evaluate the expression without using a calculator.

$\log_4 64$	$\log_9 1$	$\log_5 \sqrt{5}$
$\log_4 64$	$\log_9 1$	$\log_5 \sqrt{5}$
$\log_4 64 = x$	$\log_9 1 = x$	$\log_5 \sqrt{5} = x$
$4^x = 64$	$9^x = 1$	$5^x = \sqrt{5}$
$x = 3$	$x = 0$	$x = \frac{1}{2}$

Evaluate each logarithm below.

36) $\log_7 7$	37) $\log_3 27$	38) $\log_2\left(\frac{1}{16}\right)$
39) $\log_{25} 5$	40) $\log_8 1$	41) $\log_{27} 9$

Part 11 – Properties of Logarithms

$\log_b xy = \log_b x + \log_b y$ $\quad$ $\log_b \frac{x}{y} = \log_b x - \log_b y$ $\quad$ $\log_b x^y = y \log_b x$ $\quad$ $b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use properties of logarithms to evaluate each expression below.

42) $\log_2 2^5$	43) $\log_9 9^3$	44) $\log_2 8^3$	45) $2^{\log_2 10}$
46) $e^{\ln 8}$	47) $7 \ln e$	48) $\ln e^3$	49) $\ln \sqrt{e}$
50) $\log_{10} 25 + \log_{10} 4$		51) $\log_2 40 - \log_2 5$	

Part 12 – Factoring Trinomials

Factor the expression $2x^2 + x - 6$ using the box method.

<u>Steps to Factor Trinomials in the form $ax^2 + bx + c$ by Box Method</u>:

1) Multiply a times c. $a * c = (2)(-6) = -12$

2) Find two numbers that multiply to $a * c = -12$ **and** add up to the coefficient of the middle term $b = 1$.

$$(\mathbf{-3})(\mathbf{4}) = -12$$

$$\mathbf{-3} + \mathbf{4} = 1$$

3) Rewrite the middle term with these two numbers, put the four terms in a box, and factor the GCF out of each row and column.

$$2x^2 + \mathbf{1x} - 6$$

$$2x^2 - \mathbf{3x} + \mathbf{4x} - 6$$

	$2x$	-3
x	$2x^2$	$-3x$
$+2$	$+4x$	-6

$$2x^2 + x - 6 = (2x - 3)(x + 2)$$

Factor each expression below by using the box method.

52) $5x^2 + 14x + 8$

53) $8n^2 - 10n + 3$

54) $2x^2 - 3x - 9$

55) $6x^2 - 23x - 4$

Part 13 – The Discriminant

A quadratic equation $ax^2 + bx + c = 0$ has **discriminant** $\Delta = b^2 - 4ac$.

The symbol Δ is used to represent the discriminant.

The discriminant can be used to determine (or *discriminate*) the number and nature of the roots of the equation $ax^2 + bx + c = 0$, or the number of x-intercepts of the graph of the equation $y = ax^2 + bx + c$.

For…	**The equation $ax^2 + bx + c = 0$ has …**	**The graph of $y = ax^2 + bx + c$ has …**
$b^2 - 4ac > 0$	two distinct real roots	two x-intercepts
$b^2 - 4ac = 0$	two equal real roots (one repeated root)	one x-intercept
$b^2 - 4ac < 0$	no real roots	no x-intercepts

Use the discriminant to determine the nature of the roots of each quadratic equation below.

56) $$4x^2 - 12x + 9 = 0$$

57) $$7x^2 - x + 2 = 0$$

58) $$3x^2 + 18x = -5$$

59) $$24x^2 + 5 = 14x$$